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LIMITS OF APPLICABILITY OF THE BOHM FORMULA

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A comparison is made between the densities of ion current on spherical and cylindrical probes calculated by Bohm's approximate formula and on the basis of a rigorous numerical solution of Vlasov's system of equations.

Probe methods of diagnosing plasmas have now found wide application. If the plasma is of sufficiently low density, the concentration of charged particles can be calculated from the Bohm formula [1]

$$n_i = \frac{I_i}{ae\,(2\,kT_e/m_i)^{1/2}S}. \tag{1}$$

The coefficient a = 0.8 for a spherical probe and 0.4 for a cylindrical probe. As was noted in [2], Eq. (1) is valid if the mean free path of the particles of the plasma λ is much greater than the probe dimension r_0 and if r_0 is much greater than the thickness of the space-charge layer Δ. Also, the value of T_i of the ions must be much less than the electron temperature. The potential of the probe φ_0 must be negative and of sufficient magnitude. These conditions reduce to the following system of inequalities:

$$\lambda \gg r_0 \gg \Delta, \tag{2}$$

$$T_e \gg T_i, \tag{3}$$

$$e\varphi_0/kT_i \ll 0. \tag{4}$$

Conditions (2)-(4) are encountered in practice in measurements in a glow-discharge plasma and in low-pressure arcs if the concentration of charged particles $n_i \geqslant 10^{10}$ cm^{-3}. The validity of Eq. (1) was checked repeatedly by comparing probe measurements with measurements obtained by other independent methods. We performed one such comparison using results for a molecular plasma flow coming out of a plasmatron. We used a cylindrical probe with its axis parallel to the flow axis. The concentration of charged particles in the flow was about 10^{12} cm^{-3}. Under these conditions, the thickness of the space-charge layer proves to be much less than the probe radius. Thus, the end effect can be ignored [3]. By selecting a probe with a length much greater than its radius, we also succeeded in establishing conditions such that the directed velocity had no effect on the ion saturation current. The con-

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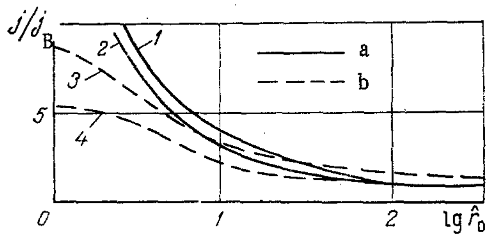

Fig. 1. Dependence of j/j_B on $\hat{r}_0$ and $\varepsilon(\hat{\varphi}_0 = -30)$: 1, 3) $\varepsilon = 1$; 2, 4) $\varepsilon = 0$; a) sphere; b) cylinder.

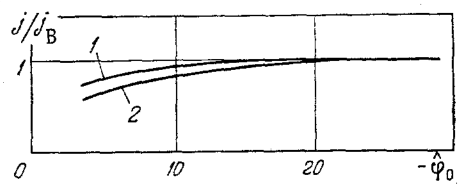

Fig. 2. Dependence of j/j_B on $\hat{\varphi}_0$ for a sphere with $\varepsilon = 0$ (curve 1) and $\varepsilon = 1$ (2). $\hat{r}_0 = 10^2$.

centration of charged particles was measured by the probe (in accordance with Eq. (1)) and, simultaneously, the method of high-frequency-signal radioscopy. The disagreement was no greater than 20%.

Experience shows that the limits of applicability of Eq. (1) for analyzing probe characteristics is broader than indicated in [2]. The investigators in [4-7] performed numerical calculations with Vlasov's system of equations in boundary layers of ionized gases and obtained families of volt—ampere characteristics for probes as a function of the determining parameters of the problem. This allowed us to compare the probe current calculated by Eq. (1) with its exact value and to thereby evaluate the range of applicability of the Bohm formula. It was shown in [6, 7] that the ion current on a spherical or cylindrical probe depends on three dimensionless parameters: $\hat{r}_0 = r_0/r_d$; $\hat{\varphi}_0 = e\varphi_0/kT_i$; $\varepsilon = T_i/T_e (r_d = (kT_i/4\pi n_i e^2)^{1/2})$. The interpretation of the probe characteristic generally depends on all three parameters, which, in particular, include the ion temperature. It is not possible to determine T_i from a classic probe characteristic [2]. It is thus first necessary to evaluate T_i theoretically or by somehow measuring it (not with a probe) and then to analyze the characteristics.† The exceptions are the cases when $T_e = T_i$ or $T_e \gg T_i$ and, thus, the parameter ε is known beforehand: it is equal to 1 or 0. The conditions of applicability of the Bohm formula, mentioned above, correspond to the case $\varepsilon = 0$. Thus, there remain only the two characteristic parameters $\hat{r}_0 = r_0/r_d^*$ and $\hat{\varphi}_0 = e\varphi_0/kT_e$ on which depends the density of the probe current $\hat{j}_i (r_d^* = r_d \varepsilon^{1/2})$.

It is clear from physical considerations that the dependence of $\hat{j}_i$ on $\hat{\varphi}_0$ should become weaker with an increase in $\hat{r}_0$. In fact, $\hat{r}_0$ may increase without limit, while the thickness of the space-charge layer is no greater than several tenths of a Debye radius in the range $-30 \leqslant \hat{\varphi}_0 \leqslant -1$. The relative change in the thickness of the space-charge layer with an increase in potential at large $\hat{r}_0$ is not great, so the probe current is practically independent of the potential.

Figure 1 compares the probe current on the basis of the rigorous theory in [4-7] and the probe current calculated by the Bohm formula (1). The ratio of these currents j/j_B is plotted off the vertical axis, while the dimensionless radius of the probe $\hat{r}_0$ is plotted off the horizontal axis. The calculations were performed with a sufficiently large negative potential for the probe $\hat{\varphi}_0 = -30$. We used ε as the design parameter. Figure 2 shows the effect of the dimensionless potential of the probe on the ratio j/j_B with a prescribed radius. The following conclusions can be made on the range of applicability of the Bohm formula from an examination of the curves.

†It was suggested in [7] that it is possible to determine T_i by means of a nonstationary Langmuir probe.

1. If it is assumed [7] that $\Delta \sim 10r_d$, then the inequality $\hat{r}_0 \gg \Delta$ is satisfied for $\hat{r}_0 \geqslant 10^2$, so that $r_0/\Delta \geqslant 10$. The exception is the case of a cylindrical probe, when $\varepsilon = 1$. Here, inequality (2) should be reinforced: $\hat{r}_0 \geqslant 10^3$.

2. It is not necessary to satisfy condition (3). Equation (1) is valid for both $T_e \gg T_i$ and $T_e = T_i$. It is valid beginning with $\hat{r}_0 > 50$ in the case $\varepsilon = 0$ and beginning with $\hat{r}_0 > 10^2$ (sphere) or $\hat{r}_0 > 10^3$ (cylinder) in the case $\varepsilon = 1$.

3. Condition (4) turns out to be satisfied if $|e\varphi_0/kT_i| > 25$. Somewhat less negative potentials can be used compared to the case $\varepsilon = 1$ (Fig. 2) when $\varepsilon = 0$.

The curves in Figs. 1 and 2 make it possible to select a characteristic probe dimension and potential if the required accuracy of the determination of the concentrations is prescribed in a probe experiment using Eq. (1).

NOTATION

n_i, concentration of ions; I_i, ion current; m_i, mass of ion; T_i, ion temperature; T_e, electron temperature; k, Boltzmann constant; e, electrode charge; S, surface of probe; r_0, radius; φ_0, potential of probe; λ, mean free path; Δ, thickness of space-charge layer; r_d, Debye radius; j, current density; j_B, current density calculated by the Bohm formula.

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HEATING OF THIN FILMS BY LASER RADIATION WITH ALLOWANCE FOR THE TEMPERATURE DEPENDENCE OF THE REFLECTION COEFFICIENT

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We solve the thermophysical problem of the heating of thin metal films on semiconductor substrates by laser radiation for linear and stepwise changes of the absorptivity as a function of the surface temperature.

The laser alloying of semiconductors is a promising method of obtaining p–n junctions. Laser radiation is focused on a semiconductor substrate of gallium arsenide [1] or silicon [2, 3] covered with a film of thickness h ∿ 300-3000 Å of the alloying metal. The calculation of the diffusion of the metal into the semiconductor requires first solving the thermal problem of the heating of a two-layer system by a laser pulse.

The reflectivity of metals depends strongly on the condition of the surface (oxide film, quality of preparation, etc.). An analysis of the experimental data for a clean polished

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